PRESIDENCY UNIVERSITY, KOLKATA

Syllabus for Three Year B.Sc. MATHEMATICS (GenEd) Course

(With effect from the Academic Session 2013-14)

Module Structure

Semester	Module No.	Name of the Module	Marks
I	M11	Differential Calculus	50
I	M12	Algebra	50
I	M13	The Joy of Numbers	50
II	M21	Integral Calculus and Differential Equations	50
II	M22	Analytical Geometry	50
II	M23	Puzzles and Paradoxes in Mathematics	50
III	M31	Linear Algebra	50
III	M32	Set Theory and Logic	50
IV	M41	Numerical Analysis	50
IV	M42	Mathematical Analysis	50

Module M11
Differential Calculus

1. Real Numbers: Axiomatic definition. Intuitive idea of completeness.
2. Real-valued functions defined on an interval : Limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval. Acquaintance (no proof) with the important properties of continuous functions on closed intervals.
3. Derivative – its geometrical and physical interpretation. Sign of derivative – Monotonic increasing and decreasing functions. Relation between continuity and differentiability.
4. Successive derivative – Leibnitz's Theorem and its application.
5. Mean Value Theorems and expansion of functions like $e^x, \sin x, \cos x.(1+x)^n, \ln(1+x)$ (with validity of regions).
6. Applications of Differential Calculus : Maxima and Minima, Tangents and Normals, Pedal equation of a curve. Definition and examples of singular points (viz. Node, Cusp, Isolated point).
7. Indeterminate Forms : L'Hospital's Rule : Statement and problems only.
8. Sequence of real numbers: convergence, Cauchy criteria and other elementary properties. Series of real number, Absolute and conditional convergence of series.

Module M12
Algebra

1. Complex Numbers: De Moivre's Theorem and its applications. Exponential, Sine, Cosine and Logarithm of a complex number. Definition of e^z, Inverse circular and Hyperbolic functions.
2. Theory of Equations: Fundamental Theorem of Algebra. Polynomials with real co-efficients: Descarte's Rule of sign and its applications. Relation between roots and co-efficients. Symmetric functions of roots, Transformations of equations. Solution of a cubic and biquadratic.
3. Introduction of Group Theory: Definition and examples. Elementary properties using definition of Group. Definition and examples of sub-group, Statement of necessary and sufficient condition, quotient group. Homomorphism and isomorphism.
4. Rings and Integral Domains: Definition and examples. Subrings and ideals. Quotient ring. Homomorphism ans isomorphism of rings.
5. Fields: Definition and examples.

Module M13
The Joy of Numbers

Elementary Number Theory. Prime Numbers. Fundamental Theorem of Algebra.

Different series of Numbers: special reference to Fibonacci series. Golden ratio, its presence in Arts and nature.

Module M21
Integral Calculus and Differential Equations

Group A (25 marks)
Integral Calculus

1. Integration of the form $\int \frac{dx}{a+bcosx}$, $\int \frac{l\sin x+p\cos x}{m\sin x+n\cos x}\ dx$ and integration of rational functions.
2. Evaluation of definite integrals.
3. Integration as the limit of a sum (with equally spaced as well as unequally spaced intervals)
4. Reduction formulae of $\int \sin^m x \cos^n x\ dx$, $\int \tan^x dx$ and $\int \frac{\sin^m x}{\cos^n x}\ dx$ and associated problems (m and n are non-negative integers).
5. Definition of Improper Integrals : Statements of (i) μ-test, (ii) Comparison test (Limit form excluded). Use of Beta and Gamma functions (convergence and important relations being assumed).
6. Applications: rectification, quadrature, finding c.g. of regular objects, volume and surface areas of solids formed by revolution of plane curve and areas.

Group B (25 marks)
Differential Equations

1. Order and solution of an ordinary differential equation (ODE) in presence of arbitrary constants. Formation of ODE.
2. First order equations : (i) Variables separable. (ii) Homogeneous equations and equations reducible to homogeneous forms. (iii) Exact equations and those reducible to such equation. (iv) Euler's and Bernoulli's equations (Linear). (v) Clairaut's Equations : General and Singular solutions.
3. Orthogonal Trajectories.
4. Second order linear equations : Second order linear differential equations with constant coefficients. Euler's Homogeneous equations.

Module M22
Analytical Geometry

Group A (25 marks)
Analytical Geometry of two dimensions

1. Transformations of Rectangular axes: Translation, Rotation and their combinations. Invariants.
2. General equation of second degree: Reduction to canonical forms and Classification.
3. Pair of straight lines: Condition that the general equation of 2nd degree may represent two straight lines. Points of intersection of two intersecting straight lines. Angle between two lines. Equation of bisectors of angles. Equation of two lines joining the origin to the points in which a line meets a second degree curve.
4. Equations of pair of tangents from an external point, chord of contact, poles and polars in case of general conic : Particular cases for Parabola, Ellipse, Circle and Hyperbola.
5. Polar equation of straight lines and circles. Polar equation of a conic referred to a focus as pole. Equation of chord joining two points. Equations of tangent and normal.

Group B (25 marks)
Analytical Geometry of three dimensions

1. Rectangular Cartesian co-ordinates: Distance between two points. Division of a line segment in a given ratio. Direction cosines and direction ratios of a straight line. Projection of a line segment on another line. Angle between two straight lines.
2. Equation of a Plane: General form. Intercept and Normal form. Angle between two planes. Signed distance of a point from a plane. Bisectors of angles between two intersecting planes.
3. Equations of Straight line: General and symmetric form. Distance of a point from a line. Shortest distance between two skew-lines. Coplanarity of two straight lines.
4. Sphere and its tangent plane.
5. Right circular cone and right circular cylinder:
6. Familiarity with conicoids.
7. Spherical and cylindrical coordinates.

Module M23
Puzzles and Paradoxes of Mathematics

Different Puzzles, application of system of linear equations and difference tables to solve puzzles. Construction of Magic squares of different order.

Some famous paradoxes.

Module M31
Linear Algebra

1. Vector (Linear) space over a field. Subspaces. Linear combinations. Linear dependence and independence of a set of vectors. Linear span. Finite dimensional vector space.

   Basis. Dimension. Replacement Theorem. Extension theorem. Deletion theorem.
2. Row Space and Column Space of a Matrix. Rank of a matrix. $\text{Rank}(AB) \leq \min(\text{Rank}A, \text{Rank}B)$.
3. System of Linear homogeneous equations: Solution space of a homogeneous system and its dimension.

   System of linear non-homogeneous equations: Necessary and sufficient condition for the consistency of the system. Method of solution of the system of equations.
4. Linear Transformation (L.T.) on Vector Spaces: Null space. Range space. Rank and Nullity, Sylvester's law of Nullity. Inverse of Linear Transformation. Non-singular Linear Transformation. Change of basis by Linear Transformation. Vector spaces of Linear Transformation.
5. Linear Transformation (L.T.) and Matrices: Matrix of a linear transformation relative to ordered bases of finite-dimensional vector spaces. Rank of L.T. = Rank of the corresponding matrix.
6. Characteristic equation of a square matrix. Eigen-value and Eigen-vector. Invariant subspace. Cayley-Hamilton Theorem. Simple properties of Eigen value and Eigen vector.

Module M32
Set Theory and Logic

1. Axiomatic set theory (ZFC). Ordered set. Introduction to ordinals and cardinals.

2. Formal logics: Statements. Symbolic representation.

   Axiomatic System. Classical propositional logic. Theorems. Proofs. Truth tables. Tautologies. Contradictions. Algebraic completeness theorem for classical propositional logic.

Module M41
Numerical Analysis

1. Numerical solution of equations: Determination of real roots: Bisection method, Regula-Falsi method, Newton-Raphson methods, their geometrical significance. Fixed point iteration method.
2. Errors in Numerical computation: Approximational and errors in numerical computation.
3. Interpolation: Problems of interpolation, Weierstrass' approximation theorem (only statement). Polynomial interpolation. Equispaced arguments. Difference table. Deduction of Newton's forward and backward interpolation formulae. General interpolation formulae: Deduction of Lagrange's interpolation formula. Divided difference.
4. Numerical Integration: Integration of Newton's interpolation formula. Newton-Cote's formula. Basic Trapezoidal and Simpson's 1/3 rd. formulae. Their composite forms.
5. Numerical solution of a system of linear equations: Gauss elimination method. Iterative method - Gauss-Seidal method.
6. Numerical solution of Ordinary Differential Equation: Basic ideas, nature of the problem. Picard and Euler methods (emphasis on the problems only).

Module M42
Mathematical Analysis

1. Properties of $\mathbb{R}$ as complete ordered field. Archimedean and density property. Decimal representation.
2. Concept of countability and uncountability: Elementary results. $\mathbb{Q}$ is countable. $\mathbb{R}$ is uncountable. If X is infinite $P(X)$, the power set of X, is uncountable.
3. Topology of $\mathbb{R}$: Neighbourhood of a point. Interior point. Accumulation point and isolated point of a subset of $\mathbb{R}$. Bolzano-Weierstrass Theorem. Derived set. Open set and closed set. Union, Intersection, Complement of open and closed sets in $\mathbb{R}$.
4. Real sequence: $f : \mathbb{N} \to \mathbb{R}$. Convergence and non convergence, Operations on limits, Sandwich rule. Monotone sequences and their convergence. Cauchy's general principle of convergence. Cauchy sequence. Nested interval theorem.

   Subsequence: Subsequential limits. Cluster points.
5. Compactness in $\mathbb{R}$: Covering by open intervals. Subcovering. Cantor's intersection theorem. Lindelöf-covering lemma. Compact sets. Heine-Borel Theorem and its converse. Sequential compactness.
6. Continuity of a function at a point and on a set: Piecewise continuous functions. Discontinuity of function at a point – types of discontinuity. Points of discontinuities of monotonic functions.
7. Functions defined on subsets of $\mathbb{R}$. Limit and continuity. Uniform continuity. Continuity on compact set. Properties of continuous functions on closed intervals. Bolzano's theorem. Intermediate-value property and allied results.
8. Infinite series of real numbers: Convergence, divergence. Cauchy's criterion of convergence. Series of arbitrary terms: Absolutely convergent and conditionally convergent series. Alternating series: Leibnitz test. Rearrangement of series through examples. Riemann's re-arrangement theorem (statement) and simple examples. Rearrangement of absolutely convergent series.