Presidency University, Kolkata
Department of Mathematics

Syllabus for M. Sc. Mathematics

PRESIDENCY UNIVERSITY, KOLKATA

Syllabus for two-year **MATHEMATICS M.Sc.** Course
(with effect from Academic Session 2017-18)

Outline of the Syllabus

Module	Topic	Marks
Semester I (250 Marks)		
M701	Algebra - I (Groups & Rings)	50
M702	Complex Analysis	50
M703	Topology - I (General Topology & Covering Spaces)	50
M791	Differential Equations & Dynamical Systems	50
M792	Graph Theory & Combinatorics	50
Semester II (250 Marks)		
M801	Linear Algebra & Modules	50
M802	Measure Theory	50
M803	Geometry - I (Differentiable Manifolds)	50
M891	Stochastic Processes	50
M892	Optimization	50
Semester III (250 Marks)		
M901	Algebra - II (Fields & Galois Theory)	50
M902	Functional Analysis	50
M903	Topolgy - II (Fundamental Groups & Homology)	50
M991	Elective Course	50
M992	Elective Course	50
Semester IV (250 Marks)		
M1001	Partial Differential Equations & Distributions	50
M1002	Geometry - II (Riemannian Geometry)	50
M1003	Number Theory	50
M1091	Elective Course	50
M1092	Elective Course	50

List of Elective Courses

Courses for Semester - III:

1. Commutative Algebra
2. Statistics & Data Science
3. Programming Languages
4. Ergodic Theory
5. Discrete Dynamical Systems
6. Financial Mathematics
7. Riemann Surfaces
8. Methods in Differential Equations
9. Lie Groups & Lie Algebras
10. Numerical Analysis

Courses for Semester IV:

1. Hamiltonian Mechanics
2. Quantum Mechanics
3. Advanced Analysis
4. Operator Theory
5. Representation Theory of Finite Groups
6. Information & Coding Theory
7. Cohomology & Characteristic Classes
8. Algebraic Geometry
9. Design & Analysis of Algorithm
10. Cryptology
11. One Semester Project (at most one per student; reserved only for students performing exceptionally well)

Detailed Syllabus

M701: Algebra - I (Groups & Rings)

1. Group Theory: Review of normal subgroups, quotient groups, and isomorphism theorems; Group actions with examples, orbits and stabilisers, class equations and applications; Lagrange's, Cayley's, Cauchy's and Sylow's theorems in the language of group actions; Symmetric and alternating groups, simplicity of A_n; Direct products and free Abelian groups; Semi-direct products; Composition series, exact sequences; Solvable and nilpotent groups.

   (Time permitting: Free groups; Free products, amalgamated free products, HNN-extensions, wreath products.)

2. Ring Theory: Review of integral domains, ideals, quotient rings and isomorphism theorems, prime and maximal ideals, product of rings, prime and maximal ideals in quotient rings and in finite products, Chinese remainder theorem, field of fractions, irreducible and prime elements, UFD, PID, ED; Polynomial rings, division algorithm, irreducibility criteria, Gauss' theorem; Noetherian rings, Hilbert's basis theorem.

Suggested Texts:

1. D.S. Dummit and R.M. Foote, Abstarct Algebra, Wiley, 2003.
2. S. Lang, Algbera, Springer, GTM.
3. T.W. Hungerford, Algebra, Springer, GTM-135, 1974.
4. N.S.Gopalakrishnan,University Algebra, Wiley, 1986
5. J.J.Rotman, An Introduction to the Theory of Groups, Springer Verlag 2002

M702: Complex Analysis

1. Holomorphic functions and the Cauchy-Riemann equations.
2. Power series, Analytic Functions, Exponential, Logarithmic and Trigonometric functions, Branch of a complex logarithm.
3. Complex integration, Goursat's theorem, Cauchy's integral formula, power series representation, zeros of an analytic function, Liouville's theorem, index of a closed curve, homotopy version of Cauchy's theorem, invariance of integrals under homotopy.
4. Analytic continuation, Morera's theorem, sequence of holomorphic functions.
5. Classification of singularities, meromorphic functions and residue calculus, Laurent series, contour integration.
6. Argument principle, Rouché's theorem, open mapping theorem, maximum modulus principle.

7. Möbius transformation, classification of Möbius transformations (elliptic, hyperbolic, parabolic), conformal mapping, Schwarz lemma, conformal automorphisms of disc, upper half plane, complex plane, Riemann sphere.

8. Space of continuous functions, normal families, Arzela-Ascorli theorem, compactness and convergence in the space of analytic functions, Montel's theorem, space of meromorphic functions, Riemann mapping theorem.

9. (Optional) Infinite product and Weierstrass factorization theorem.

10. (Optional) Little Picard theorem and Great Picard theorem.

Suggested Texts:

1. Functions of One Complex Variable by J B Conway.

2. Complex Analysis by E M Stein & R Shakarchi.

3. Complex Analysis by L V Ahlfors.

4. Complex Analysis by T W Gamelin.

5. Real and Complex Analysis by W Rudin.

6. Complex Analysis the Geometric Viewpoint by S G Krantz.

M703: Topology - I (General Topology and Fundamental Groups)

1. Topological spaces, open and closed sets, basis, limit points, closure, interior and boundary, subspace topology, product topology; Continuous maps: properties and constructions; pasting lemma, homeomorphisms, weak topology; metric topology.

2. Connected spaces, path-connected and locally connected spaces; compact spaces, compactness and continuity: properties, limit-point compactness, locally compact spaces, one-point compactification. (Time permitting: Lebesgue number lemma).

3. Countability axioms; Separation axioms: Hausdorff, regular and normal spaces, their relationship and properties; Urysohn lemma, Urysohn metrization theorem, Tietze extension theorem, Tychonoff's theorem (and applications of them); (Time permitting: Stone-Cech compactification, paracompactness and partitions of unity.)

4. Completion of metric spaces, Baire Category Theorem and applications. (Time permitting: (topology of) point-wise and compact convergence in function spaces, Ascoli-Arzela theorem.)

5. Quotient topology and identification spaces; topological manifolds: as examples of quotient topology - torus, Klein's bottle, projective spaces. Paths and homotopy, fundamental groups (π_1), covering spaces, simply connected spaces, universal cover, lifting properties, deck-transformations and properly discontinuous group actions, computation of $\pi_1(\mathbb{S}^1)$.

Suggested Texts:

1. Topology by J. Munkers
2. Topology by J. Dugundji
3. Introduction to Topology and Modern Analysis by G F Simmons
4. Topology by J L Kelley

M791: Differential Equations & Dynamical Systems

1. Function Space Preliminaries, The Fundamental Theorem of Existence and Uniqueness, Gronwall's Inequality, Dependence on Initial Conditions and Parameters. Maximal Interval of Existence.
2. Vector Fields and Flows on $\mathbb{R}^n$. Topological Equivalence and Conjugacy, Linear System on $\mathbb{R}^n$.
3. α & ω Limit Sets of an Orbit, Attractors, Periodic Orbits and Limit Cycles.
4. Local Structure of Critical Points (The Local Stable Manifold Theorem, The Hartman Grobman Theorem, The Center Manifold Theorem), The Normal Form Theory, Lyapunov Function, Local Structure of Periodic Orbits (The Poincaré map and Floquet Theory), The Poincaré Benedixson Theorem.
5. The Poincare Sphere and the Behavior at Infinity. Global Flow of a Polynomial Vector Field.
6. Indices of Planar Critical Points, The Poincaré Index Formula, The Poincaré Hopf Index Theorem. Higher Dimension: The Degree.
7. Structural Stability of Vector Fields, Piexoto's Theorem.
8. Chain Recurrent Set, Chain Transitivity and Morse Decomposition.

Suggested Texts:

1. L. Perko:Differential Equations and Dynamical Systems
2. C. Robinson: Dynamical Systems: Stability, Symbolic Dynamics and Chaos
3. J. Palais & W. de Melo: Geometric Theory of Dynamical Systems: An Introduction
4. V.I. Arnold: Geometrical Methods in the Theory of Ordinary Differential Equations
5. P. Glendenning, Stability, Instability and Chaos: An Introduction to the Theory of Nonlinear Differential Equations.

M792: Graph Theory & Combinatorics

1. Graphs and Digraphs; Intersection Graphs, Operations on Graphs; Connectedness, Trees, Spanning Tree; Degree Sequences: Havel-Hakimi Theorem (Statement only) and its Applications; Connectivity: Vertex and Edge Connectivity; Eulerian and Hamiltonian graphs: Necessary and Sufficient Conditions; Clique Number, Chromatic Number: Their Relations and Brooke's Theorem, Planar Graphs and 5-Colour Theorem; Domination number, Independence number: Relations and Bounds. Isomorphism of Graphs: Vertex Transitive and Edge Transitive Graphs, Cayley Graphs, Strongly Regular Graphs: Paley Graphs; Adjacency Matrix, Incidence Matrix of a Graph: Properties and Eigen Values;

2. Pigeon-Hole Principle, Inclusion-Exclusion, Ramsey's theorem, Recurrence Relations and Generating Functions.

Suggested Texts:

1. C. Godsil and G. Royle, Algebraic Graph Theory, Springer-Verlag, 2000.

2. N. Biggs, Algebraic Graph Theory, Cambridge University Press, 1974.

3. T. Haynes, S.T. Hedetniemi, P. Slater, Fundamentals of Domination in Graphs, Marcel Dekker Inc., 1998.

4. A.E. Brouwer and W.H. Haemers, Spectra of Graphs, Universitext, Springer, 2012.

5. C. Godsil, K. Meagher, Erdos-Ko-Rado Theorems: Algebraic Approaches, Cambridge University Press, 2016.

6. D.B. West, Introduction to Graph Theory, 2001.

M801: Linear Algebra & Modules

1. Quick review Linear Algebra: Vector spaces, linear transformation, matrix of a linear transform, Dual space and double dual, transpose of a linear transform, rank of a linear transform, determinants, characteristic and minimal polynomial, Cayley-Hamilton theorem, eigen values and eigen vectors, diagonalisation.

2. Inner-product spaces, Gram-Schmidt orthogonalisation, bi-linear forms, definition of unitary, hermitian, normal, real symmetric and orthogonal linear transformations, spectral theorems; multi-linear forms, alternating forms.

3. Modules over commutative rings, examples: vector spaces, commutative rings, $\mathbb{Z}$-modules, $F[X]$-modules; submodules. Quotient modules, homomorphisms, isomorphism theorems, $Hom_R(M, N)$ for R-modules M, N, generators and relations for modules, direct products and direct sums, direct summands, free modules, finitely generated modules.

4. Tensor product of modules: definition, universal property, 'extension of scalars', basic properties and elementary computations.

5. Exact sequences of modules: Projective, injective and flat modules (only definitions and examples).

6. Noetherian modules, torsion and annihilator submodules, finitely generated modules over PID, structure theorems for modules over PID: existence (invariant factor form & elementary divisor form) and uniqueness, primary decomposition theorem.

7. Applications: (a) to modules over $\mathbb{Z}$: fundamental theorem of finitely generated abelian groups; (b) to modules over $F[X]$: Canonical forms - Rational and Jordan canonical forms.

8. (Time permitting: Tensor algebras, symmetric algebras and exterior algebras).

Suggested Texts:

1. K. Hoffman and R. Kunze : Linear Algebra

2. Dummit and Foote : Abstract Algebra

3. Hungerford : Algebra

4. C. Musili : Rings and Modules

5. Larry C. Grove : Algebra

6. Larry C. Grove : Classical Groups and Geometric Algebra

M802: Measure Theory

1. Algebra, σ-algebra, Monotone Class Theorem, Measure Spaces.

2. Outer Measures, Caratheodory Extension Theorem, Pre-measures, Hahn-Kolmogorov Extension Theorem, Uniqueness of the Extension, Completion of a Measure Space.

3. Lebesgue Measure and Its Properties.

4. Measurable Functions and Their Properties, Modes of Convergence.

5. Integration, Monotone Convergence Theorem, Fatou's Lemma, Dominated Convergence Theorem.

6. Product Measures, Fubini's Theorem.

7. L^p spaces, Riesz-Fisher Theorem, Hölder's Inequality, Minkowski's Inequality, Dual of L^p spaces, Reisz Representation Theorem.

8. Absolute Continuity of Measures, Lebesgue Decomposition Theorem, Radon Nikodym Theorem & Its Applications.

9. Fundamental Theorem of Calculus for Lebesgue Integrals.

Suggested Texts:

1. An Introduction to Measure Theory by T. Tao

2. Measure and Integration by I K Rana

3. Measure Theory by P R Halmos
4. Real Analysis by H L Royden
5. Real and Complex Analysis by W. Rudin

M803: Geometry - I (Differentiable Manifolds)

1. Review of multivariate calculus in $\mathbb{R}^n$; partial, directional and total derivatives; Jacobian matrix, chain rule; inverse and implicit function theorems.
2. Review of smooth surfaces in $\mathbb{R}^3$: surfaces as level sets; tangent vetors and tangent spaces, vector fields and integral curves on surface, normal vector fields and orientation.
3. Definition of topological manifolds; definition of smooth manifolds; examples with computations of C^∞-structure; Lie groups: definition and examples, smooth partition of unity.
4. Smooth functions on manifolds, smooth maps between manifolds, diffeomorphisms; tangent vectors, tangent spaces; derivatives of smooth maps between manifolds, Jacobian matrix, rank of the Jacobian matrix.
5. Regular value and critical value of a smooth map, Sard's theorem, Morse functions, Morse Lemma; immersions, submersions, embeddings, tubular neighbourhoods; submanifolds, examples, inverse image of a regular value as a submanifold, (Time permitting: Whitney's theorem).
6. Tangent bundle, smooth vector fields, integral curves, flow of a vector field, one-parameter subgroup of transformations, Lie bracket of vector fields, f-related vector fields, Lie algebra of vector fields.
7. Review of multilinear algebra: co-variant and contravariant tensors, tensor product, wedge product, alternating tensors, smooth tensor fields.
8. Co-tangent bundle, smooth one-forms, differential k-forms, exterior product of differential forms, volume form, graded algebra structure of differential forms; operators on differential forms: exterior derivatives, contraction operator, Lie derivative; closed and exact forms, Poincaré Lemma; integration of forms, Stoke's Theorem.

Suggested Texts:

1. An Introduction to Manifolds - L. Tu, Springer.
2. An Introduction to Smooth Manifolds - J.M. Lee, Springer.
3. Foundations of Differentiable Manifolds and Lie Groups - F.W. Warner, Springer.
4. Geometry of Differential Forms - S. Morita, AMS.
5. Differential Topology - V. Guillemin, A. Pollack, AMS.

6. Notes on Differential Geometry - N.J. Hicks, van Nostrand.

7. Foundations of Differential Geometry, Vol. 1 - S. Kobayashi and K. Nomizu, Wiley.

M891: Stochastic Processes

1. Review of Probability Theory: Probability Space, Random Variables and Their Distributions.

2. Definition and Properties of Stochastic Processes.

3. Discrete Time Markov Chains, Examples, Classification of States, Stationary Distribution, Random Walk on Finite Groups, Branching Process.

4. Continuous time Markov Chains, Birth and Death Processes.

5. Optimal Stopping of Markov Chains.

6. Brownian Motions.

7. Martingales, Examples, Optional Sampling Theorem, Uniform Integrability, Martingale Convergence Theorem.

Suggested Texts:

1. Stochastic Process by S M Ross

2. Stochastic Processes with Applications by R N Bhattacharya & E C Waymire

3. Introduction to Stochastic Process by P G Hoel, S C Port & C J Stone

M892: Optimization

1. Revised simplex algorithm, bounded-variables algorithm, parametric linear programming, duality & post-optmal analysis.

2. Goal Programming, Integer Programming, Dynamic Programming: Algorithms & Applications.

3. Transportation & Network Flow Problems: Simplex method for Transportation problems, The Assignment problem, Network concepts, Minimum cost flow, Maximal flow.

4. Unconstrained Problems: Examples, First-order & Second-order conditions, Minimization & Maximization of convex functions, Descent method, Newton Raphson Method, Davidon-Fletcher-Powell method.

5. Constrained Problems: Examples, First-order & Second-order conditions, Inequality constraints, Zero-order conditions & Lagrange multipliers, Primal methods, Penalty & Barrier methods, Dual & Cutting plane methods.

Suggested Texts:

1. Nonlinear Programming, M. S. Bazaraa, H. D. Sherali and C. M. Shetty, John Wiley & Sons.
2. Nonlinear Programming, O. L. Mangasarian, R. R. Meyer and Johnson (Eds.) Academic Press, New York.
3. An Introduction to Optimization, Chang, K. P. Edwin and Stanislaw Zak, John Wiley & Sons Inc.
4. Operations Research, H A Taha, Prentice-Hall

M901: Algebra - II (Field & Galois Theory)

1. Field Theory: Field extensions, finite and algebraic extensions, algebraic closure, splitting fields, normal extensions, separable, inseparable and purely inseparable extensions, finite fields, ruler and compass constructions.
2. Galois theory: Galois extensions and Galois groups, fundamental theorem of Galois theory; Examples, explicit computation and applications of Galois theory; Roots of unity, cyclotomic extensions, construction of regular n-gons, solvability by radicals, quintics are not solvable by radicals.

Suggested Texts:

1. David A. Cox : Galois Theory
2. Dummit and Foote : Abstract Algebra
3. Ian Stewart : Galois Theory
4. Serge Lang : Algebra
5. Hungerford : Algebra
6. Joseph Rotman : Galois Theory

M902: Functional Analysis

1. Normed linear spaces, Banach spaces and examples, quotient space and completeness, locally-convex topological vector spaces, bounded linear operators, compact operators.
2. Hahn-Banach theorem, some consequences; review of Baire category theorem, uniform boundedness principle, Banach-Steinhaus theorem, applications; open mapping and closed graph theorems, applications.
3. Dual of a Banach space, computing duals of L^p, $(1 \leq p < \infty)$ and $C[0, 1]$; reflexive spaces; weak and weak* topologies, Banach-Alaoglu theorem, Krein-Milman theorem.
4. Hilbert spaces, orthogonal sets, projection theorem, Bessel's inequality, complete orthonormal sets and Parseval's identity. Riesz representation theorem.

5. Operators on a Hilbert space: examples and basic properties, algebra of bounded linear operators, adjoint of an operator, self-adjoint operators, projections, compact operators, normal and unitary operators, diagonalization of compact self-adjoint operators; Spectrum and spectral radius, spectral theorem for compact self-adjoint operators.

6. (Time Permitting: Spectral theorem for normal and unitary operators).

Suggested Texts:

1. A course in Functional Analysis-J.B.Conway.

2. Functional Analysis-Walter Rudin.

3. Functional Analysis- Kosaku Yosida.

4. Functional Analysis- B.V.Limaye.

5. Notes on Functional Analysis- R.Bhatia.

M903: Topology - II (Fundamental Groups & Homology)

1. Review of fundamental groups and universal covering spaces, van Kampens theorem, fundamental groups of closed surfaces Σ_g of genus g.

2. Simplicial complexes, chains and boundary homomorphisms, simplicial homology, examples and computations.

3. Singular homology; categories, functors and Eilenberg-Steenrod axioms; H_1 as the abelianisation of π_1 (explicit illustration through $\pi_1(\Sigma_g)$ and $H_1(\Sigma_g)$); equivalence of simplicial and singular homology; statement of universal coefficient theorem for homology.

4. Relative homology, excision and exact sequences with applications; Mayer-Vietoris sequence and applications; homotopoy invariance (Time permitting: Homology with coefficients).

5. CW-complexes, cellular homology, computing cellular homology for CW-complexes like $\mathbb{S}^n$, $\mathbb{RP}^n$, $\mathbb{CP}^n$, lens spaces L, closed surfaces Σ_g, etc.; Betti numbers and Euler characteristics.

6. (Time permitting: Singular cohomology, statement of the universal coefficient theorem, cup and cap products, Künneth formula, Cohomology ring).

Suggested Texts:

1. Algebraic Topology: A First Course - Greenberg, Harper, CRC Press.

2. Topology and Geometry - Bredon, Springer.

3. Algebraic Topology - A. Hatcher, CUP.

M1001: Partial Differential Equations & Distributions

1. Initial & Boundary Value Problems, Well-posedness.
2. First Order PDEs, The Method of Characteristics, Shocks & Traffic Dynamics.
3. Classification of Second Order PDEs.
4. The Heat Equation, Existence and Uniqueness of Solutions, Fundamental Solutions and Green's Function, Duhamel's Method.
5. The Laplace/Poisson Equation, Existence and Uniqueness of Solutions, Fundamental Solutions and Green's Function, Harmonic Functions, Mean Value Theorem, Maximum Principle, Smoothness of Solutions, Liouville's Theorem, Representation Formulae for Solutions of Drichlet and Neumann Problems.
6. The Wave Equation, Type of Waves, Uniqueness of Solutions, D'Alembert's Formula, Duhamel's Method, Kirchoff's Formula.
7. Non-linear Partial Differential Equations. Conversion to Linear Forms. Travelling Waves. Burgers Equation. Dimensional Analysis and Similarity. Nonlinear Diffusion and Dispersion. KdV, Nonlinear Schrdinger and Sine-Gordon Equations. Backlund Transformations. Inverse Scattering Method
8. Test Functions, Distributions, Examples, Application to PDEs.
9. Introduction to Sobolev Spaces and Regularity Theory for Elliptic Equations.

Suggested Texts:

1. L.C. Evans, Partial Differential Equations, AMS
2. I.N. Sneddon, Elements of Partial Differential Equation, McGraw-Hill
3. D. Mitrea, Distributions, Partial Differential Equations and Harmonic Analysis, Springer
4. P.J. Olver, Introduction to Partial Differential Equations, Springer
5. G.B. Folland, Introduction to Partial Differential Equations, Princeton University Press
6. Q. Han, A Basic Course in Partial Differential Equations, AMS
7. P. Prasad & R. Ravirandran, Partial Differential Equations, New Age International

M1002: Geometry - II (Riemannian Geometry)

1. Review of the geometry of surfaces in $\mathbb{R}^3$: Geodesic on a surface, Gauss map, covariant derivative of vector fields along a curve, parallel vector fields, Weingarten map and its self-adjointness, normal curvature, principal curvatures, Gauss and mean curvature, Gauss-Bonnet theorem.

2. Riemannian metric, examples: $\mathbb{R}^n$, $\mathbb{S}^n$, $\mathbb{H}^n$, proof of existence; Riemannian length, volume element; induced Riemannian structures: immersed (sub)manifolds, submanifolds, Riemannian products, Riemannian covering maps; Bi-invariant Riemannian metric on a Lie group, homogeneous Riemannian space, Riemannian symmetric space, examples.

3. Affine connections, co-variant derivative along a curve, parallel transport; Riemannian (or Levi-Civita) connection, proof of existence, Riemannian connection on a submanifold.

4. Geodesics, existence and uniqueness, exponential maps, Gauss' lemma, normal neighborhood; Riemannian manifold as a metric space, complete Riemannian manifolds and Hopf-Rinow theorem; Geodesic flow, geodesic flows on $\mathbb{S}^2$, $\mathbb{H}^2$. (Time permitting: Killing field).

5. Riemannian curvature tensor, sectional curvature, Ricci and scalar curvature, computations.

Suggested Texts:

1. Riemannian Geometry - M. do Carmo, Birkhauser.

2. Riemannian Geometry - P. Petersen, Springer.

3. Riemannian Geometry - S. Gallot, D. Hulin, J. Lafontaine, Springer.

4. Notes on Differential Geometry - N.J. Hicks, van Nostrand.

M1003: Number Theory

1. Simple continued fractions, infinite continued fractions and irrational numbers, periodic continued fractions, algorithms for solving Brahmagupta-Pell equation, numerical computations.

2. Revision of unique factorization, congruences, chinese remainder theorem; algebraic number fields and their ring of integers, units and primes, factorisation, quadratic and cyclotomic fields, Diophantine equations, structure of $U(\mathbb{Z}/n\mathbb{Z})$.

3. Quadratic reciprocity: quadratic residues, quadratic Gauss sums, Gaussian reciprocity law, Jacobi symbol; finite fields, equations over finite fields, Gauss and Jacobi sums, cubic and biquadratic reciprocity.

4. Riemann zeta function, prime number theorem, Dirichlet's L-function, elliptic curves.

Suggested Texts:

1. Saban Alaca, Kenneth S Williams : Introduction to Algebraic Number Theory, Cambridge University Press.

2. I.Niven,H.S.Zuckerman and H.Montgomery, An Introduction to the Theory of Numbers, John Wiley

3. A Friendly Introduction to Number Theory, J.H. Silverman, Prentice-Hall

Elective Courses

Design & Analysis of Algorithm

1. *Introduction and basic concepts:* Complexity measures, Master's Theorem, worst-case and average-case complexity functions, problem complexity, quick review of basic data structures and algorithm design principles.

2. *Sorting and selection:* Finding maximum and minimum, k largest elements in order; Sorting by selection, tournament and heap sort methods, lower bound for sorting, other sorting algorithms-radix sort, quick sort, merge sort; Selection of k-th largest element.

3. *Searching and set manipulation:* Searching in static table-binary search, path lengths in binary trees and applications, optimality of binary search in worst cast and average-case, binary search trees, construction of optimal weighted binary search trees; Searching in dynamic table-randomly grown binary search trees, AVL and (a, b) trees.

4. *Hashing:* Basic ingredients, analysis of hashing with chaining and with open addressing.

5. *Union-Find problem:* Tree representation of a set, weighted union and path compression-analysis and applications.

6. *Graph problems:* Graph searching -BFS, DFS, shortest first search, topological sort; connected and biconnected components; minimum spanning trees-Kruskal's and Prim's algorithms-Johnson's implementation of Prim's algorithm using priority queue data structures.

7. *Algebraic problems:* Evaluation of polynomials with or without preprocessing. Winograd's and Strassen's matrix multiplication algorithms and applications to related problems, FFT, simple lower bound results.

8. *String processing:* String searching and Pattern matching, Knuth-Morris-Pratt algorithm and its analysis.

9. *NP-completeness:* Informal concepts of deterministic and nondeterministic algorithms, P and NP, NP-completeness, statement of Cook's theorem, some standard NP-complete problems, approximation algorithms.

Statistics & Data Science

1. Multiple linear regression; least square estimation and properties of least squares residuals; Inference with multiple linear regression;

2. Lack of fit: Graphical diagnostics and tests.

3. Regression with errors in the predictors; Collinearity: consequences, diagnostics and strategies (including ridge & shrinkage regression, LASSO, dimension reduction methods).

4. Generalized Least Squares; Weighted Least Squares;

5. Transforming the Response and the predictors; model building; subset-selection methods: stepwise regression; AIC and BIC methods. Principal Components; Ridge regression; Shrinkage methods: Principal components, Ridge regression, LASSO. Log-Linear models.

6. Introduction to Generalized Linear Models (GLMs), illustration with logit and probit analysis. Linear predictor, link function, canonical link function, deviance. Maximum likelihood estimation using iteratively re-weighted least square algorithm. Goodness of fit test.

Programming Languages

1. *Introduction:* Overview of different programming paradigms e. g. imperative, object oriented, functional, logic and concurrent programming.

2. *Syntax and semantics of programming languages:* An overview of syntax specification and semiformal semantic specification using attribute grammar.

3. *Imperative and Object Oriented Languages:* Names, their scope, life and binding. Control-flow, control abstraction; in subprogram and exception handling. Primitive and constructed data types, data abstraction, inheritance, type checking and polymorphism.

4. *Functional Languages:* Typed-calculus (Lambda calculus), higher order functions and types, evaluation strategies, type checking, implementation.

5. *Logic Programming:* Computing with relation, first-order logic, SLD-resolution, unification, sequencing of control, negation, implementation, case study.

6. *Concurrency:* Communication and synchronization, shared memory and message passing, safety and liveness properties, multithreaded program.

7. *Formal Semantics:* Operational, denotational and axiomatic semantics, languages with higher order constructs and types, recursive type, subtype, semantics of nondeterminism and concurrency.

8. *Assignments:* Using one or more of the following as based on time constraints: C++/Java/OCAML/Lisp/Haskell/Prolog.

Representation Theory of Finite Groups

1. Modules and Representations : Review of Elementary Module Theory, Basic Notions of The Representation Theory of Finite Groups, Maschke'S Theorem.

2. Wedderburn'S Classification of Semisimple Algebras and Its Consequences.

3. Character Theory of Finite Groups : Characters and Their Elementary Properties, Character Table, Computation of Character Table, Importance of Characters in Representation Theory.

4. Representations over Complex Numbers.

5. Representations of Symmetric Groups : The Rsk Correspondence, Semistandard Young Tableaux, Classification of Simple Representations of Symmetric Groups.

6. Representations of Alternating Groups.

Suggested Texts:

1. Alperin and Bell : Groups and Representations

2. Amritanshu Prasad : Representation Theory (A Combinatorial Viewpoint)

3. William Fulton and Joe Harris : Representation Theory (A First Course)

Discrete Dynamical Systems

1. One dimensional dynamics:

   - Examples of discrete dynamical systems, iterations of functions, phase portraits, periodic points and stable sets, differentiability and its implications, graphical analysis, cobweb diagram.
   - Quadratic family, symbolic dynamics, Topological conjugacy, chaos, structural stability, Sarkovskii's theorem, Schwarzian derivative, bifurcation theory.
   - Newton's method and its applications.
   - Circle maps, rotation number, periodic points of circle maps, Poincaré classification theorem, devil's staircase, Denjoy example, Morse-Smale diffeomorphisms.
   - Homoclinic points and bifurcations, period doubling route to chaos, kneading theory.

2. Higher dimensional dynamics:

   - Dynamics of linear maps, the horseshoe map, hyperbolic toral automorphisms, attractors, stable and unstable manifold theorem, global results and hyperbolic sets, the Hopf bifurcation, the Hénon map.

3. Complex Analytic Dynamics:

   - Normal families, exceptional points, fixed points and periodic points (attracting, repelling, indifferent), Parabolic fixed points and the Fatou flower, Julia set and Fatou set, quadratic maps in $\mathbb{C}$, Mandelbrot set.

Suggested Texts:

1. An Introduction to Chaotic Dynamical Systems by R L Devaney.

2. A First Course in Discrete Dynamical Systems by R A Holmgren.

3. A First Course in Dynamics with a Panorama of Recent Developments by B Hasselblatt & A Katok.
4. Chaos An Introduction to Dynamical Systems by K T Aligood, T D Sauer, J A Yorke.
5. Iterations of Rational Functions: Complex Analytic Dynamical Systems by A F Beardon.
6. Dynamics in One Complex Variable by J W Milnor.
7. An Introduction to Dynamical Systems: Continuous and Discrete by R C Robinson.
8. Introduction to the Modern Theory of Dynamical Systems by A Katok & B Hasselblatt.

Financial Mathematics

Syllabus:

1. The Binomial asset pricing model: No-arbitrage pricing model, one-period binomial model, multiperiod binomial model, computational considerations, change of measure and Radon-Nikodym derivative process, risk-neutral pricing of European derivative securities.
2. Stochastic Integral : The Itô stochastic integral for simple processes, the general Itô stochastic integral, the Itô lemma, the Stratonovich integral.
3. Stochastic Differential Equations : Itô stochastic differential equations, Solving Itô stochastic differential equations by Itô lemma and also via Stratonovich calculus, general linear differential equations with additive noise and with multiplicative noise, the expectation and variance functions of the solution, solution via numerical techniques (Euler approximation, Milstein approximation).
4. Applications of stochastic calculus in finance: Options, mathematical formulation of option pricing problem, the Black-Scholes option pricing formula.

Cryptology

1. Introduction: Historical ciphers and their cryptanalysis, adversarial models and principles ofdefining security.
2. Perfectly-Secret Encryption: Definitions, the one-time pad and its limitations.
3. Private-Key (Symmetric) Encryption: Computational security, defining secure encryption, constructing secure encryption, pseudorandomness, chosen plaintext attacks (CPA), constructing CPA-secure encryption, modes of operation, CBC versus CTR, Chosen ciphertext attacks.
4. Message Authentication Codes: Message integrity, definition of security, constructions from pseudorandom functions, CBC-MAC, authenticated encryption.

5. Collision-Resistant Hash Functions: Definitions, Merkle-Damgard transform, HMAC, Birthday attacks, the Random oracle model, password hashing.

6. Number Theory: Primes, factoring and RSA, cryptographic assumptions in cyclic groups, collision resistant hash functions from discrete log.

7. Public-Key (Asymmetric) Cryptography: Diffie-Hellman key exchange, the model and definitions, hybrid encryption and KEM/DEM, El Gamal cryptosystem, RSA cryptosystem, textbook encryption, attacks on textbook RSA, padded RSA, CCA-secure RSA KEM.

8. Digital Signatures: Definition, Hash and sign, RSA signatures: textbook RSA, hashed RSA, security with ROM, certificates and public-key infrastructures, SSL/TLS.

9. Secret Sharing: Shamir secret sharing scheme and its applications.

Methods in Differential Equations

1. Strum Liouville Theory, Self-adjoint ODEs, Hermitian Operators, Green's Function, Eigenfunction Expansion.

2. Fourier Series: Periodic Extensions, The Pointwise and Uniform Convergence of Fourier Series, Differentiation and Integration of Fourier Series.

3. Integral Transforms: Fourier Integral and Fourier Transforms, Inverse Fourier Transforms. Fourier Transform of Derivatives, Convolution Theorem. Laplace Transforms, Inverse Laplace Transforms, Laplace Transform of Derivatives, Convolution Theorem. Solution of Differential Equations Using Integral Transforms.

4. The Method of Separation of Variables, The Method of characteristics & Other Standard Methods to Solve PDEs.

5. Green's Function for One Dimensional Boundary Value Problems and Planar Poisson Equation.

6. Series Solution Method of Frobenius, Legendre, Bessel, Hermite and Hypergeometric Functions.

7. Integral Equations, Integral Transforms, Generating Functions, Neumann Series, Seperable Kernels, Hilbert Schmidt Theory.

8. Calculus of Variations, The Brachistochrone Problem, Minimal Surfaces, Variation with Constraints, Euler Lagrange Equations, Variable-endpoint Conditions, Broken Extremals.

Suggested Texts:

1. G.F. Simmons: Differential Equation with Applications and Historical Notes.

2. A.C. King, J. Billingham and S.R. Otto: Differential Equations: Linear, Nonlinear, Ordinary, Partial.

3. I. N. Sneddon: Elements of Partial Differential equations
4. I. N. Sneddon: Use of Integral Transforms
5. Francesco G. Tricomi: Integral Equations
6. Gelfand and Fomin: Calculus of Variations
7. G. P. Tolstov: Fourier Series

Advanced Analysis

1. The vibrating string, The wave equation, The heat equation.
2. Fourier Series, Uniqueness, Convolutions, Good Kernels, Ceŝaro and Abel summability: Application to Fourier Series.
3. Mean-square convergence of Fourier series, Pointwise convergernce.
4. The isoperimetric inequality, Weyls equidistribution theorem, A continuous but nowhere differentiable function.
5. Elementary theory of the Fourier transform, The Schwartz space, The Fourier transform on $\mathcal{S}$, The Fourier inversion, The Plancherel formula, The Poisson summation formula, The Heisenberg uncertainty principle.
6. The Fourier Transform on $\mathbb{R}^d$, Symmetries, Integration, The wave equation in $\mathbb{R}^d \times \mathbb{R}$.
7. (Optional) Fourier analysis on $\mathbb{Z}(N)$, Fourier analysis on finite abelian groups.

Numerical Analysis

1. Numerical Solution of System of Linear Equations: Triangular Factorisation Methods, Matrix Inversion Method, Operation Counts, Iterative Methods Jacobi Method, Convergence Condition Of Gauss-Seidel Method and Gauss-Jacobi Method, Importance of Diagonal Dominance.
2. Non-Linear Systems of Equations: Newtons Method, Quasi-Newtons Method.
3. Numerical Integration: Problem of Approximate Quadrature, Trapezoidal & Simpson's Rule with Error Formula, Newton-Cotes Formulae, Richardson Extrapolation, Romberg Integration, Double Integrals Cubature Formula of Simpson Type, Improper Integrals.
4. Numerical Solution of Initial Value Problems for ODE:
   First Order Equation: Runge-Kutta Methods, Multistep Predictor-Corrector Methods Adams-Bashforth Method, Adams-Moulton Method, Milnes Method, Convergence and Stability. Higher Order Equations: Rkf4 Method.

5. Numerical Solution of PDE By Finite Difference Methods: Parabolic Equation in One Dimension (Heat Equation), Explicit Finite Difference Method, Implicit Crank-Nicolson Method, Hyperbolic Equation in One-Space Dimension (Wave Equation) Finite Difference Method, Method of Characteristics (Consistency, Convergence and Stability).

6. Algorithms & Programming for Numerical Methods in C (or C++).

Suggested Texts:

1. R. L. Burden, D. Faires, A. M. Burden: Numerical Analysis

2. F. B. Hildebrand:Introduction to Numerical Analysis

3. Ralstone and Rabinowitz: First Course in Numerical Analysis

4. Dahlquist and Bjorck: Numerical Methods

Quantum Mechanics

1. The Basic Concepts of Quantum Mechanics. The Uncertainty Principle, Complimentarity & Duality. The Wave Function & Measurements.

2. Matrix Formulation of Quantum Mechanics. Operators, Representation of Operators.

3. Schrodinger's Wave Equation, Probability Current Density, Ehrenfest's Theorem, Energy Eigenfunctions. The Variational Principle.

4. One Dimensional Problems. Scattering Theory. The Harmonic Oscillator.

5. Three Dimensional Problems. Symmetries in Quantum Mechanics. Angular Momentum Operators. Spherical Harmonics, Parity.

6. The Spin Operator, The Wave Function of Particles with Spin. Identical Particles, Pauli's Exclusion Principle.

7. The Hydrogen Atom.

8. Stationary Perturbation Theory, Stark and Zeeman Effects.

Suggested Texts:

1. L.I. Schiff, Quantum Mechanics.

2. J.J. Sakurai, Modern Quantum Mechanics.

3. L. D. Landau and E. M. Lifshitz, Quantum Mechanics: Non-relativistic Theory, Course of Theoretical Physics Vol 3, Pergamon Press Ltd (1958).

4. D. J. Griffiths, Introduction to Quantum Mechanics

Hamiltonian Mechanics

1. Lagrangian Mechanics on Manifolds, E Noether's theorem, D' Alembert's Principle.
2. Symplectic Vector Spaces, Symplectic Manifolds and Symplectic Morphisms, Darboux's Theorem, The Cotangent Bundle.
3. Hamiltonian Vector Fields and Poisson Brackets.
4. Hamiltonian Phase Flow and Their Integral Invariants.
5. Huygen's Principle.
6. The Hamilton Jacobi Method and Generating Functions.

Suggested Texts:

1. G. Vilasi: Hamiltonian Dynamics
2. Lanczos: The variational principles of mechanics
3. Goldstein: Classical Mechanics
4. V. I. Arnold: Mathematical methods in classical mechanics

Lie Groups & Lie Algebras

1. Lie Groups and Their Lie Algebras : Definitions and Examples, Homomorphisms, Lie Subgroups, Covering Group of a Lie Group, Covering Homomorphism, Simply Connected Lie Groups, Exponential Map and Its Properties, Exponential Maps for $GL_n(\mathbb{C})$ and Its Consequences in Determination of Subgroups of $GL_n(\mathbb{C})$, Continuous Homomorphisms, Closed Subgroups, The Adjoint Representation, Homogeneous Manifolds, Examples of Homogeneous Manifolds, Connectedness of Classical Lie Groups.
2. Lie Algebras : Solvable and Nilpotent Lie Algebras, Theorems of Lie and Engel, Killing Form, Semisimple Lie Algebras, Compact Lie Algebras.
3. Structure Theory of Complex Semisimple Lie Algebras : Cartan Subalgebra, Root Space Decomposition, Real Forms, Compact Real Form. Examples, The Classical Complex Lie Algebras. Cartan Decomposition.

Suggested Texts:

1. Frank W. Warner : Foundations of Differential Manifolds and Lie Groups
2. Sigurdur Helgason : Differential Geometry, Lie Groups and Symmetric Spaces
3. James E. Humphreys : Introduction to Lie Algebras and Representation Theory

Operator Theory

1. Banach algebra of linear operators, elementary properties and examples, ideals, homomorphisms and quotients, spectrum of a linear operator, spectral radius formula, commutative Banach algebra, Gelfand transformation.

2. $\mathcal{B}(H)$: strong and weak operator topology, $\mathcal{B}(L^2(\mu))$ and M_ϕ.

3. C^*-algebras: examples and basic properties, approximate identitites, commutative C^*-algebras, Gelfand-Neumark theorem, spectral mapping theorem, representations of C^*-algebras and the GNS-construction.

4. von Neumann Algebras: projections, double commutant theorem, L^∞-functional calculus.

5. Normal operators, spectral theorem: 'bounded normal operators are unitarily equivalent to M_ϕ' (outline of the proof).

6. Compact operators on Hilbert spaces: Fredholm Theory, Index.

7. (Optional) Toeplitz operators.

Suggested Texts:

1. C^*-algebras and operator theory- Gerard.J. Murphy.

2. An introduction to operator algebras- Kehe Zhu.

3. Operator algebras: Theory of C^*-algebras and Von-Neumann algebras- Bruce Blackadar.

Ergodic Theory

1. Measure preserving systems; examples: Hamiltonian dynamics and Liouville's theorem, Bernoulli shifts, Markov shifts, rotations of the circle, rotations of the torus, automorphisms of the Torus, Gauss transformations, skew-product.

2. Poincaré recurrence lemma, induced transformation, Kakutani towers, Rokhlins lemma; Recurrence in topological dynamics, Birkhoffs recurrence theorem.

3. Ergodicity, weak-mixing and strong-mixing and their characterisations, ergodic theorems of Birkhoff and von Neumann, consequences of the ergodic theorem, invariant measures on compact systems, unique ergodicity and equidistribution, Weyl's theorem.

4. The isomorphism problem; conjugacy, spectral equivalence.

5. Transformations with discrete spectrum, Halmos-von Neumann theorem.

6. Entropy, the Kolmogorov-Sinai theorem, calculation of entropy, the Shannon-McMillan-Breiman theorem.

7. Flows, Birkhoff's ergodic theorem and Wiener's ergodic theorem for flows, flows built under a function.

Suggested Texts:

1. PeterWalters, An introduction to ergodic theory, Springer-Verlag (1982).

2. M. G. Nadkarni, Basic ergodic theory, TRIM 6, Hindustan Book Agency (1995).

3. K. Petersen, Ergodic theory, Cambridge Studies inAdvancedMathematics (2), Cambridge University Press (1989).

Information & Coding Theory

1. Introduction to information theory: Random variables, Probability distributions, Entropy of a discrete random variable and its properties, entropy of discrete random vectors and mutual information, entropy of continuous random variables and their properties.

2. Capacity-cost function of a discrete memoryless channel: Definitions and properties of capacitycost function, the channel coding theorem.

3. Rate-distortion function of a discrete memoryless source: Definitions and properties of ratedistortion function, the source coding theorem.

4. The source-channel coding theorem.

5. Introduction to coding theory: Linear codes, generator and parity check matrices, syndrome decoding, Hamming codes, weight enumerators.

6. Cyclic codes: Definitions and properties, shift-register encoders, cyclic Hamming codes, bursterror correction.

7. BCH, Reed-Solomon and Goppa codes: Definitions, examples, properties, applications.

Cohomology & Characteristic Classes

1. Singular cohomology: co-chains and co-boundary map, reduced cohomology groups, long exact sequence of pairs and relative cohomology groups, induced homomorphisms, homotopy invariance, excision; Axioms for cohomology, statement for universal coefficient theorem for cohomology.

2. Simplicial cohomology, cellular cohomology and computations; Cup product, cap product and cohomology rings; cross product and statement of the Künneth formulas, computations of the cohomology rings of the spaces: $\mathbb{RP}^n$, $\mathbb{CP}^n$, $\mathbb{RP}^\infty$, $\mathbb{CP}^\infty$, etc.

3. Orientation on a compact manifold, fundamental class, Poincaré duality, Alexander duality, examples and computations.

4. Review of smooth differential forms and exterior differentiation, graded algebra of smooth differential forms on a compact manifold, de Rham cohomology, Poincaré duality.

Algebraic Geometry

1. Varieties: Affine and projective varieties, coordinate rings, morphisms and rational maps, local ring of a point, function fields, dimension of a variety; Nöether normalisation lemma, Hilbert Nullstellensatz.

2. Curves: Singular points and tangent lines, multiplicities and local rings, intersection multiplicities, Bezout's theorem for plane curves, Max Noether's theorem and some of its applications, group law on a nonsingular cubic, rational parametrization, branches and valuations; Divisors on curves, Abelian differential, Riemann Roch for curves.

Suggested Texts:

1. W. Fulton,Algebraic curves. An introduction to algebraic geometry,Addison-Wesley (1989).

2. C.G. Gibson, Elementary Geometry of Algebraic Curves, Cambridge.

3. I.R. Shafarevich, Basic algebraic geometry, Springer.

4. J. Harris, Algebraic geometry. A first course, GTM (133), Springer-Verlag (1995).

5. C. Musili, Algebraic geometry for beginners, TRIM (20), HBA (2001).

6. B. Hassett, Introduction to Algebraic Geometry, Cambridge University Press (2007)

Commutative Algebra

1. Prime avoidance; Nilradical and Jacobson radical; extension and contraction of ideals under ring homomorphisms; .

2. Free modules; Projective Modules; Tensor Product of Modules and Algebras; Flat, Faithfully Flat and Finitely Presented Modules; Shanuels Lemma.

3. Localisation and local rings, universal property of localisation, extended and contracted ideals and prime ideals under localisation, localisation and quotients, exacteness property. Results on prime ideals like theorems of Cohen and Isaac. Nagatas criterion for UFD and applications; equivalence of PID and one-dimensional UFD.

4. Modules over local rings. Cayley-Hamilton, NAK lemma and applications. Examples of local- global principles. Projective and locally free modules. Patching up of Localisation.

5. Polynomial and Power Series Rings. Noetherian Rings andModules. Hilberts Basis Theorem. Associated Primes and Primary Decomposition. Artininan Modules. Modules of Finite Length.

6. Integral Extensions: integral closure, normalisation and normal rings. Cohen-Seidenberg Going-Up Theorem. Hilberts Nullstellensatz and applications.

7. Valuations, Discrete Valuation Rings, Dedekind domains.

Suggested Texts:

1. N.S.Gopalakrishnan: Commutative Algebra
2. M.F. Atiyah & I. G. Macdonald : Introduction to Commutative Algebra
3. M. Reid: Undergrdauate Commutative Algebra
4. E.. Kunz: Introduction to Commutative Algebra and Algebraic Geometry

Riemann Surfaces

1. The idea of a Riemann Surface, Simple Examples of Riemann Surfaces, Riemann Surfaces of algebraic functions, Riemann Surface structure on Cylinder and Torus.
2. Metrics on Riemann Surfaces, Triangulations of Compact Riemann Surfaces, Discrete Groups of Hyperbolic Isometries: Fundamental Polygons, Topological Classification of Compact Riemann Surfaces.
3. Functions on Riemann Surfaces, Holomorphic functions, Singularities, Meromorphic functions, Laurent Series, Order of a meromorphic function, Theorems from one complex variable, Meromorphic functions on Riemann sphere and torus, Elliptic function, Weierstrass $\wp$-function.
4. Holomorphic maps between Riemann Surfaces, Isomorphisms and automorphisms, Local Normal Form and Multiplicity, Degree of a Holomorphic map between compact Riemann Surfaces, Sum of orders of meromorphic function, The Euler number of a compact surface, Riemann Hurwitz formula.
5. Group Actions of Riemann Surfaces, Stabilizer subgroups, The quotient Riemann Surfaces, The Ramification of the Quotient map, Hurwitz's Theorem and Automorphisms.
6. Monodromy Theorem.
7. (Optional) Forms on Riemann Surfaces, Operations on Forms, The Poincaré Dolbeault Lemmas, Integration on Riemann Surfaces, Stoke's Theorem, Residue Theorem, Homotopy and Homology.

Suggested Texts:

1. Algebraic Curves and Riemann Surfaces by R Miranda.
2. Riemann Surfaces by H M Farkas & I Kra.
3. Complex Functions: An Algebraic and Geometric Viewpoint by G A Jones & D Singerman.
4. Compact Riemann Surfaces by J Jost.
5. Compact Riemann Surfaces by R Narasimhan.
6. Lectures in Riemann Surfaces by O Forster.